Evaluation of Ratio between Splitting Tensile Strength and Compressive Strength for Concretes up to 120 MPa and its Application in Strength Criterion

by Nihal Arıoglu, Z. Canan Girgin, and Ergin Arıoglu

INTRODUCTION

The ratio between tensile strength and compressive strength is an important material property of concrete. The value of this ratio is required for the following applications:

1. With respect to Bortolotti’s studies, the ultimate strain value in uniaxial tension is expressed in terms of this strength ratio.

2. According to Johnston’s strength criterion for intact rock under triaxial compression, the material constants defining the failure envelope are related to the ratio of compressive strength to tensile strength. Reported results of Setunge et al. and Yapi Merkezi for very high-strength concrete in triaxial compression are in good agreement with the strength criterion proposed by Johnston.

3. There are three types of tests to measure strength in tension: direct tension, flexure, and splitting tension. It has been well established that the simplest and the most reliable method, which generally provides a lower coefficient of variation, is the splitting tensile test of a cylindrical specimen. In this test, a cylindrical specimen is loaded in compression diametrically between two plates. According to the theory of elasticity, this loading generates almost uniform tensile stress along the diameter, which causes the specimen to fail by splitting along a vertical plane. The splitting strength can be used to estimate direct tensile strength by multiplying by a conversion factor of 0.9, as given in the CEB-FIB Code and by Hannant et al.

The objectives of the investigation reported herein are as follows:

1. To evaluate the ratio of splitting tensile strength to compressive strength as a function of cylinder compressive strength of concrete by means of regression analysis of experimental data from the literature.

2. To verify whether Johnston’s strength criterion is valid for high-strength concretes by making use of the derived relationship between the ratio of splitting tensile strength to compressive strength and the cylinder compressive strength in this study.

RESEARCH SIGNIFICANCE

This study introduces a relationship between the ratio of splitting tensile strength to compressive strength and the cylinder compressive strength, which is applicable to concrete at early ages (12 hours and longer) as well as very high-strength concrete (up to 120 MPa [17,400 psi]). Existing relationships in the literature are based mainly on data obtained from concretes with compressive strength of not more than 83 MPa (12,000 psi). The reliability of the proposed equation is assessed based on integral absolute error (%). The results of this analysis are particularly important because no comprehensive information on the reliabilities of the relationships used in the current building codes has been available.

In the design of triaxially compressed structures, it is necessary to have an expression relating the ultimate strength and the confining pressure. Johnston proposed an empirical strength criterion based on the ratio of compressive to tensile strength and the confinement effectiveness, for a range of geomaterials. This study shows that Johnston’s strength criterion can be used to adequately predict the ultimate strength of very high-strength concrete under triaxial compression. Knowledge of the ratio between splitting tensile strength and uniaxial compressive strength should allow for the estimation of strength of very high-strength concrete under confinement. Furthermore, this knowledge could reduce costs associated with triaxial testing programs for very high-strength concrete.

Keywords: compressive strength; confinement; high-strength concrete; splitting tensile strength; stress.
RESULTS OF REGRESSION ANALYSIS AND DISCUSSION

General
In recent studies, the failure envelope for very high-strength concrete subjected to confining pressure was shown to be in reasonable agreement with Johnston’s strength criterion. If accurate estimates for very high-strength concrete under confinement are required, the relationship between the strength ratio and the compressive strength must be established by means of the regression analysis. The following factors must be taken into account in this analysis:

1. The mathematical model should be based on physically significant parameters. Furthermore, it should be as simple as possible and easily usable in any analysis.
2. The relationship should be applicable over a wide range of experimental data.
3. The coefficient of correlation $r$ that measures the strength of the proposed relationship should be large. It is important to note that even when the correlation is significant, the variability can still be large, and the proposed equation may not be reliable.
4. The accuracy of the relationship should be as high as possible. In other words, the errors associated with the regression model should be as small as possible.

Table 1—Brief descriptions of main data—221 test data points—used for regression analysis

<table>
<thead>
<tr>
<th>Mixture properties</th>
<th>Gardner¹¹</th>
<th>Gardner et al.¹²</th>
<th>Imam et al.¹³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixture numbers</td>
<td>6</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>Cement type</td>
<td>Type I, II</td>
<td>Type III</td>
<td>P 50*</td>
</tr>
<tr>
<td>Cement quantity, kg/m³</td>
<td>225 to 409 (Type I)</td>
<td>304 to 411</td>
<td>410 to 550</td>
</tr>
<tr>
<td>Fly ash, kg/m³</td>
<td>75,101 (two mixtures)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Silica fume, kg/m³</td>
<td>—</td>
<td>—</td>
<td>0 to 82.5</td>
</tr>
<tr>
<td>Total cementitious materials, kg/m³</td>
<td>300 to 414</td>
<td>304 to 411</td>
<td>410 to 632.5</td>
</tr>
<tr>
<td>Coarse aggregate, kg/m³</td>
<td>1040 to 1080†</td>
<td>1039 to 1058 (gravel), 1039 to 1058, crushed limestone ($D_{max} = 25$ mm)</td>
<td>0 to 1097 (gravel), 0 to 1375 (porphyry)</td>
</tr>
<tr>
<td>Sand, kg/m³</td>
<td>780 to 878</td>
<td>780 to 832 (natural sand)</td>
<td>430 to 720</td>
</tr>
<tr>
<td>Water, kg/m³</td>
<td>138 to 169</td>
<td>143 to 169</td>
<td>119 to 152</td>
</tr>
<tr>
<td>High-range water-reducing admixture, kg/m³</td>
<td>0 to 2.1 (N)</td>
<td>2.85 (for $w/c$ = 0.35) (N), N/A (for others)</td>
<td>12.3 to 22.1</td>
</tr>
<tr>
<td>Air-entraining agent, mL/m³</td>
<td>—</td>
<td>360 to 493</td>
<td>—</td>
</tr>
<tr>
<td>Physical properties after mixing and molding</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density, kg/m³</td>
<td>—</td>
<td>2343 to 2435</td>
<td>2370 to 2465</td>
</tr>
<tr>
<td>Slump, mm</td>
<td>50 to 100</td>
<td>38 to 64</td>
<td>10 to 240</td>
</tr>
<tr>
<td>Air content, %</td>
<td>N/A (five mixtures)</td>
<td>4.0 to 5.8</td>
<td>—</td>
</tr>
<tr>
<td>Ratios (by mass)</td>
<td>0.25 (two mixtures)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Fly ash/cement + fly ash</td>
<td>0 (four mixtures)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Silica fume/cement + silica fume</td>
<td>—</td>
<td>—</td>
<td>0 to 13%</td>
</tr>
<tr>
<td>$w/c$</td>
<td>0.55, 0.35 (each one is three mixtures)</td>
<td>0.55, 0.45, 0.35 (each one is two mixtures)</td>
<td>0.24 to 0.29</td>
</tr>
<tr>
<td>Total aggregate/cementitious materials</td>
<td>4.5 to 6.2</td>
<td>4.5 to 6.1</td>
<td>3.0 to 3.8</td>
</tr>
<tr>
<td>Curing conditions</td>
<td>Each mixture cured in water tanks at 0, 10, 20, and 30 °C</td>
<td>Three mixtures cured in seawater at 0 °C, three mixtures cured in moist chamber at 22 °C</td>
<td>Fog room 20 ± 2 °C, 95 ± 2% RH</td>
</tr>
<tr>
<td>Testing time</td>
<td>1 to 112 days</td>
<td>3 to 360 days</td>
<td>28 days</td>
</tr>
</tbody>
</table>

Compressive strength, MPa (psi) 4.0 to 56.7 (573 to 8223) 13.4 to 69.9 (1943 to 10,138) 82 to 117.4 (11,893 to 17,027) 13.4 to 69.9 (1943 to 10,138) 82 to 117.4 (11,893 to 17,027)

Splitting tensile strength, MPa (psi) 0.77 to 4.92 (111 to 713) 2.1 to 5.83 (304 to 845) 5.94 to 7.74 (861 to 1122)

Note: N = naphthalene-based; 1 kg = 2.205 lb; 1 kg/m³ = 1.685 lb/yd³; 1 L = 0.220 gal.; and 1 MPa = 145.038 psi.

¹Portland cement with compressive strength $\geq 50$ MPa at 28 days according to EN 196.
²Saturated surface dry.
³Compressive strength and splitting tensile strength values are average of five specimens.

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In this study, the reliability of the relationships derived from the regression analysis was assessed on the basis of the integral absolute error (IAE, %). This index has been used by others\textsuperscript{11,19,20} to evaluate the goodness of fit of proposed relationships, and it is computed from Eq. (1)

\[
\text{IAE} = \sum \frac{(O_i - P_i)^2}{\sum O_i} \cdot 100
\]

where \(O_i\) is the observed value, and \(P_i\) is the predicted value from the regression equation. The IAE measures the relative deviations of data from the regression equation. When the IAE is zero, the predicted values from the regression equation equal to the observed values; this situation rarely occurs. When comparing different equations, the regression equation having the smallest value of the IAE can be judged as the most reliable. A range of the IAE from 0 to 10% may be regarded as the limits for an acceptable regression equation.

**Brief presentation of experimental data**

The main sources of data used for the regression analysis along with the characteristics of the concretes (type of cement, cement quantity, \(w/c\) ratio, type of supplementary cementitious material, curing temperatures, and testing age) are given in Table 1. As seen from Table 1, the data\textsuperscript{11-13} collected from the literature is representative of the diversity that may occur in concrete construction. The cylinder compressive strength \(f_c\) varies from approximately 4 to 120 MPa (580 to 17,400 psi). In other words, the regression analysis was based on data ranging from immature concrete to very high-strength concrete.

**Results of regression analyses**

To evaluate the ratio of splitting tensile strength to compressive strength \(f_{tsp}/f_c\), a series of regression analyses was undertaken and the results of these analyses are summarized in Table 2. The values of the IAE computed for the regression equations given in Table 2 are compiled in Table 3. From Table 2 and 3, the following observations can be made:

1. Based on the coefficient of correlation \(r\), Eq. (4) to (7) provide equally strong relationships between the ratio of splitting tensile to compressive strength \(f_{tsp}/f_c\) and compressive strength \(f_c\). In terms of relative error, Eq. (6) and (7) provide the smallest values of IAE. Thus these are several equations with similar reliability. The power function, Eq. (7), can be selected for its simplicity without loss of accuracy.

2. Using the control data, the value of IAE for Eq. (7) was found to be 7.14%. Equation (7) is also in a close agreement with the control test data ranging between 6 to 122 MPa (870 to 17,690 psi). In brief, Eq. (7) shows a better accuracy for predicting the splitting tensile strength.

**Fig. 1—Ratio of splitting tensile to compressive strength versus cylinder compressive strength:** (a) group I used to obtain \(A\) and \(B\); and (b) compared with verification data II.

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**Table 2—Results of several statistical models used in regression analyses**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Statistical model</th>
<th>(A)</th>
<th>(B)</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) (f_{tsp}/f_c = A f_c/B + f_c)</td>
<td>0.08559</td>
<td>-4.3450</td>
<td>0.838</td>
<td></td>
</tr>
<tr>
<td>(3) (\log f_{tsp}/f_c = A f_c + B)</td>
<td>-0.003707</td>
<td>-0.8338</td>
<td>0.888</td>
<td></td>
</tr>
<tr>
<td>(4) (f_{tsp}/f_c = 1/A + B f_c)</td>
<td>6.2739</td>
<td>0.08814</td>
<td>0.967</td>
<td></td>
</tr>
<tr>
<td>(5) (f_{tsp}/f_c = A f_c + B)</td>
<td>0.3898</td>
<td>0.03756</td>
<td>0.951</td>
<td></td>
</tr>
<tr>
<td>(6) (f_{tsp}/f_c = A f_c^n)</td>
<td>0.3870</td>
<td>-0.3700</td>
<td>0.951</td>
<td></td>
</tr>
<tr>
<td>(7) (f_{tsp}/f_c = A f_c^n)</td>
<td>0.3870</td>
<td>-0.3700</td>
<td>0.951</td>
<td></td>
</tr>
</tbody>
</table>

Note: \(A, B =\) constants of regression equation; \(r =\) correlation coefficient of regression equation; and \(n = 221\), corresponding to group (I).

**Table 3—Calculated values of IAE for equations given in Table 2**

<table>
<thead>
<tr>
<th>Equation</th>
<th>IAE, % (I)</th>
<th>IAE, % (II)</th>
<th>IAE, % (III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2)</td>
<td>22.81</td>
<td>12.45</td>
<td>19.60</td>
</tr>
<tr>
<td>(3)</td>
<td>7.34</td>
<td>8.58</td>
<td>7.72</td>
</tr>
<tr>
<td>(4)</td>
<td>6.37</td>
<td>7.49</td>
<td>6.72</td>
</tr>
<tr>
<td>(5)</td>
<td>5.97</td>
<td>7.48</td>
<td>6.44</td>
</tr>
<tr>
<td>(6)</td>
<td>5.55</td>
<td>7.67</td>
<td>6.21</td>
</tr>
<tr>
<td>(7)</td>
<td>5.60</td>
<td>7.14</td>
<td>6.07</td>
</tr>
</tbody>
</table>

Note: IAE calculated for: (I) one group data used for regression analysis (\(n = 221\)) (range: 4 to 118 MPa [580 to 11,115 psi]);\textsuperscript{11-13} (II) other group data used as “control data” to assess accuracy of derived regression equations (\(n = 104\)) (range: 6 to 122 MPa [870 to 17,690 psi]);\textsuperscript{3,14-17} This group of data was selected randomly; (III) \(= (I) + (II)\) corresponding to all data (\(n = 325\)) (range: 4 to 122 MPa [580 to 17,690 psi]);\textsuperscript{3,14-17} where IAE equals integral absolute error, \(n\) equals numbers of data test points, and 1 MPa = 145.038 psi.
Discussion

As shown in Fig. 1(a), the ratio of the two strengths \( f_{\text{sp}}/f_c \) is strongly affected by the level of the compressive strength \( f_c \). This ratio decreases with increasing compressive strength at a decreasing rate. This finding can be explained by the fact that the increase in the splitting tensile strength \( f_{\text{sp}} \) occurs at a much smaller rate compared to the increase of compressive strength. The result is in agreement with various researchers.\(^{21-24}\) From Fig. 1(a) and (b), it is also evident that, in comparison with normal-strength concrete (NSC), at higher strengths (80 to 120 MPa [11,600 to 17,400 psi]—very high-strength concrete) there is a significant decrease in the ratio. For example, the ratio of \( f_{\text{sp}}/f_c \) varies between 0.15 and 0.10 for the NSC, while the same ratio is between 0.08 and 0.06 for very high-strength concrete. This finding implies that for the compressive strengths above approximately 100 MPa (14,000 psi), there is no further increase in the tensile strength.\(^{25}\) According to the results published by Komloš,\(^{24}\) the ratio between the splitting tensile strength and compressive strength of 200 mm cubes was found to be 0.092 and 0.067 for 7- and 180-day specimens \( (w/c = 0.4) \), respectively. The examined ratio reached a value of approximately 0.06 after 360 days of curing for the same mixture. Also, the result of this study agrees with Komloš.\(^{24}\)

As previously stated, there is little information in the literature concerning the accuracy and validity of the equations used for the purpose of estimating splitting tensile strength from compressive strength. This is especially true for very high-strength concretes. To assess the accuracy of other power function relationships, which are provided in Table 4, the IAE concept was used for the experimental data reported by the various researchers\(^{11,19,20}\) within the range 4 to 120 MPa (580 to 17,400 psi) as well as in intervals of 20 MPa (2900 psi).

On close examination of Table 4, the following findings can be obtained:

1. Based on the values of IAE calculated, the splitting tensile strength of concrete is not proportional to the square root of compressive strength. This is particularly true for \( f_{\text{sp}} > 40 \) MPa (5800 psi). The ACI models\(^{26,27}\) underestimate the splitting tensile strength for concrete with compressive strength \( f_c > 40 \) MPa (5800 psi). The same findings were mentioned previously by other investigators.\(^{8,21,23}\)

2. In the case of the CEB-FIB equation,\(^{10}\) the value of IAE varies between 2.5 and 8.9%. When all ranges are considered, the IAE is computed as 5.9%. It is interesting to note that although the equation in question is based on \( f_c < 83 \) MPa

### Table 4—Calculated integral absolute errors for several relationships (splitting tensile strength, cylinder compressive strength) in terms of strength range

<table>
<thead>
<tr>
<th>Source/Relationship</th>
<th>IAE, %</th>
<th>Compressive strength, MPa (psi)</th>
<th>Range, MPa</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 to 20</td>
<td>20 to 40</td>
<td>40 to 60</td>
</tr>
<tr>
<td>ACI 363R-92(^{26})</td>
<td>( f_{\text{sp}} = 0.59f_c^{0.5} )</td>
<td>14.4</td>
<td>5.8</td>
<td>9.7</td>
</tr>
<tr>
<td>ACI 318-99(^{23})</td>
<td>( f_{\text{sp}} = 0.56f_c^{0.5} )</td>
<td>10.9</td>
<td>8.6</td>
<td>14.0</td>
</tr>
<tr>
<td>CEB-FIB(^{10})</td>
<td>( f_{\text{sp}} = 0.3f_c^{2/3} )</td>
<td>8.9</td>
<td>6.0</td>
<td>5.6</td>
</tr>
<tr>
<td>Mokhtarzadeh and French(^{28})</td>
<td>( f_{\text{sp}} = 0.56f_c^{0.5} )</td>
<td>10.8</td>
<td>8.6</td>
<td>14.0</td>
</tr>
<tr>
<td>Carino and Lew(^{29})</td>
<td>( f_{\text{sp}} = 0.32f_c^{0.63} )</td>
<td>15.4</td>
<td>17.4</td>
<td>18.9</td>
</tr>
<tr>
<td>Raphael(^{30})</td>
<td>( f_{\text{sp}} = 0.313f_c^{0.667} )</td>
<td>12.4</td>
<td>8.3</td>
<td>7.1</td>
</tr>
<tr>
<td>Ahmad and Shah(^{31})</td>
<td>( f_{\text{sp}} = 0.462f_c^{0.55} )</td>
<td>9.3</td>
<td>10.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Gardner et al(^{12})</td>
<td>( f_{\text{sp}} = 0.47f_c^{0.59} )</td>
<td>13.8</td>
<td>7.3</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>( f_{\text{sp}} = 0.46f_c^{0.60} )</td>
<td>14.1</td>
<td>8.1</td>
<td>5.1</td>
</tr>
<tr>
<td>Gardner(^{41})</td>
<td>( f_{\text{sp}} = 0.34f_c^{0.66} )</td>
<td>8.8</td>
<td>5.8</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>( f_{\text{sp}} = 0.33f_c^{2/3} )</td>
<td>8.9</td>
<td>6.0</td>
<td>5.6</td>
</tr>
<tr>
<td>Oluokun et al(^{8})</td>
<td>( f_{\text{sp}} = 0.294f_c^{0.69} )</td>
<td>10.9</td>
<td>7.7</td>
<td>7.1</td>
</tr>
<tr>
<td>Arıoglu(^{32})</td>
<td>( f_{\text{sp}} = 0.321f_c^{0.661} )</td>
<td>10.0</td>
<td>8.5</td>
<td>8.8</td>
</tr>
<tr>
<td>Current study</td>
<td>( IAE = 0.387f_c^{-0.37} )</td>
<td>9.0</td>
<td>5.6</td>
<td>4.8</td>
</tr>
</tbody>
</table>

\(^{*}\) Including control data used.

Note: 1 MPa = 145.038 psi.
(12,035 psi), it can be extrapolated to higher strengths without any loss of accuracy.

3. The equations reported by Gardner et al., Gardner,11 and Oluokun et al.8 which were derived originally for normal concrete strengths, yield reasonable errors for high strengths.

4. The proposed model in the present study can be regarded as a realistic representation, which is applicable to concrete at early ages as well as to very high-strength concrete up to 120 MPa (17,400 psi) (refer to Fig. 1(a)). To further verify the proposed equation, the experimental data (n = 104), which were not used in the regression analysis, compared with the regression equation, as shown in Fig. 1(b). These data are shown to be in a close agreement with Eq. (7).

**APPLICABILITY OF JOHNSTON’S STRENGTH CRITERION**

Johnston3 proposed an empirical criterion to predict the compressive strength for intact geomaterials under confinement. The criterion in question can be expressed by the following equation (Fig. 2)

\[ \frac{f_1}{f_c} = \left( 1 + \frac{M}{B} \cdot \frac{f_{ftsp}}{f_c} \right)^B \]  

(8)

\[ \lambda = 0.9 \] CEB-FIB10.

As discussed by Johnston,3 and as supported by limited experimental evidence,3 this ratio is not a constant but seems to change with not only the rock type, but also the rock strength.

The material constants (M, B) in Johnston’s strength criterion can be determined from a regression analysis, taking into account a series of triaxial tests on intact samples of rock or concrete. If there are no laboratory triaxial compression test data, the values of the constants can be estimated by the following two equations

\[ B = 1 - 0.0172(\log 1000f_c)^2 \]  

(9)

\[ M = B \frac{f_c}{f_{fr}} = B \frac{f_c}{k f_{ftsp}} \]  

(10)

for a wide variety of geomaterials:3 0.08 MPa ≤ f_c ≤ 600 MPa

in which \( f_c \) is the uniaxial compressive strength, in MPa; and \( f_{ftsp} \) is the splitting tensile strength, in MPa. The value of \( f_c/f_{ftsp} \) can be estimated from the proposed Eq. (7) in Table 2 and Fig. 1. The value \( \lambda \) is the factor for converting splitting tensile strength to direct tensile strength and is assumed to equal 0.9 (CEB-FIB10).

It is worthwhile to mention that the value of \( B \) depends strongly on material strength. When \( B = 1 \), as in the case of normally consolidated soils, Johnston’s strength criterion

**Table 5—Comparison of Johnston’s strength criterion with other existing strength criteria for very high-strength concretes**

<table>
<thead>
<tr>
<th>Source</th>
<th>Uniaxial compressive strength range ( f_c ), MPa</th>
<th>Maximum strength ratio ( f_1/f_c )</th>
<th>Confinement ratio ( f_{fr}/f_c )</th>
<th>Equation</th>
<th>No. of data</th>
<th>IAE* %</th>
<th>IAE† %</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xie et al.15</td>
<td>60 to 119</td>
<td>0.82 to 3.21</td>
<td>0 to 0.504</td>
<td>[ \frac{f_1}{f_c} = \left( 1 + k \frac{f_{fr}}{f_c} \right)^B ]</td>
<td>33</td>
<td>4.7</td>
<td>7.2</td>
<td>Silica fume concrete with ( 0.216 ) to 0.321</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[ k = 21.2 - 0.05f_c ]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attard and Setunge16</td>
<td>60 to 132</td>
<td>1.042 to 2.417</td>
<td>0.004 to 0.25</td>
<td>[ \frac{f_1}{f_c} = \left( 1 + \frac{f_{fr}}{f_c} \right)^n ]</td>
<td>24</td>
<td>3.7</td>
<td>7.9</td>
<td>Silica fume concrete ( f_{fr} = 0.62 f_c ) (100 to 132 MPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[ n = 0.9 f_{ftsp} ]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[ k = 1.25 \left( 1 + 0.062 f_{fr} f_{ftsp} \right)^{0.21} ]</td>
<td>14</td>
<td>6.1</td>
<td></td>
<td>Concrete without silica fume ( f_{fr} = 0.32 f_c ) (60 to 126 MPa)</td>
</tr>
</tbody>
</table>

*IAE = integral absolute error for existing empirical strength equations concerning experimental triaxial compressive data by researchers reported in this table.
†IAE = integral absolute error for Johnston’s strength criterion concerning experimental data reported by researchers reported in this table (the ratio of splitting tensile strength to cylinder compressive strength is predicted by proposed Eq. (7) in this study).

Note: \( f_1 \) = ultimate compressive strength under triaxial compression; \( f_{fr} \) = confining pressure; \( k \) = constant in empirical strength criterion equation; \( f_c \) = uniaxial cylinder compressive strength; \( f_{ftsp} \) = splitting tensile strength; \( f_{fr} \) = direct tensile strength of concrete; and 1 MPa = 145.038 psi.
simplifies to the Mohr-Coulomb criterion. In brief, $B$ defines the nonlinearity of the failure envelope (Fig. 2) and is a measure of the confinement effectiveness.

The experimental results reported by Xie et al.\textsuperscript{15} and by Attard and Setunge\textsuperscript{16} for high-strength concretes were compared with Johnston’s strength criterion. The comparisons are displayed in Table 5. As seen from the table, the failure envelopes based on Johnston’s strength criterion, for which only one parameter (cylinder compressive strength) is needed, have prediction errors (IAE) varying between 7.2 and 7.9%. Such an error level can be regarded acceptable for practical engineering applications. The failure envelopes given by the various researchers\textsuperscript{15,16} result in smaller prediction errors (IAE = 3.7 to 6.1%) because these failure expressions were based on the specific experimental data.

Figure 3 shows a comparison of the computed axial compressive strengths $f_1$ from Johnston’s strength criterion and two triaxial compression data sets\textsuperscript{15,16} (the comparison was done in terms of the normalized strengths: $f_1/\sigma_c; f_1/\sigma_c$). From Fig. 3, it can be seen that the estimates obtained from Johnston’s strength criterion for high-strength concretes are in good agreement with the failure envelopes obtained by the researchers.\textsuperscript{15,16} The linear failure envelope ($f_1 = f_c + 4.1f_c$) proposed by Richart et al.\textsuperscript{33} underestimates the triaxial compressive concrete because their equation was based on tests of low-strength concrete ($f_c = 32$ MPa).

In the following section, a numerical example will demonstrate how to use Johnston’s strength criterion for evaluation of the strength of high-strength concrete under triaxial compression.

![Fig. 3—Experimental data of concretes under triaxial compression compared with strength criteria proposed by various researchers.](image)

**NUMERICAL EXAMPLE**

The following experimental results were obtained from a study\textsuperscript{16} carried out on concrete subjected to triaxial compression.

- **Cylinder compressive strength:** $f_c = 60$ MPa (8700 psi) ($w/cm = 0.45$).
- **Applied confining pressure:** $f_t = 10$ MPa (1450 psi).

The following steps are used to compute the ultimate axial compressive strength $f_1$ and the benefit of confinement for the given confining pressure $f_t$:

- **Estimate the splitting tensile strength to cylinder compressive strength ratio:**
  \[ \frac{f_{tsp}}{f_c} = 0.387 \]
  \[ B = 0.9 \times 0.085 = 0.076 \]

- **Estimate the material constants ($M, B$) corresponding to Johnston’s strength criterion:**
  \[ f_1 = \frac{1}{B} \left( f_c + 0.607 \right) \]
  \[ M = \frac{f_c}{f_1} = \frac{f_c}{0.9f_{tsp}} \]
  \[ B = \left( \frac{1 + M}{B} \right) f_1 \]
  \[ f_1 = \frac{f_1}{f_c} = \left( \frac{1 + M}{B} \right) f_1 \]
  \[ f_1 = 2.017f_c = 2.017 \times 60 = 121.0 \text{ MPa} (17,550 \text{ psi}) \]

- **Evaluation of the above result:**
  According to Attard and Setunge’s study,\textsuperscript{16} the axial compressive strength $f_1$ was determined to be 122 MPa (17,694 psi) (p. 435, Table 3) for the given confining pressure. Thus, the calculated ultimate compressive strength is in excellent agreement with the experimental finding.

For $f_t = 10$ MPa (1450 psi), the benefit of confinement can be evaluated by means of the efficiency ratio:

\[ \text{efficiency ratio} \% = \frac{f_1}{f_c} \times 100 = \frac{121}{60} \times 100 \approx 200\% \]

As can be seen, the axial compressive strength $f_1$ increases two-fold due to a modest confinement stress. Moreover, for a given confining pressure $f_t$, the equations published by Balmer,\textsuperscript{34} in which normal and shear stresses are related to principal stresses, can be used to estimate the angle of friction $\phi$, the failure angle $\alpha$, and the cohesive strength $C$.

**CONCLUSIONS**

The following conclusions can be drawn from this study:

1. A simple power function is proposed to evaluate the ratio of the splitting tensile to compressive strength ($f_{tsp}/f_c$) as a function of the cylinder compressive strength $f_c$ (Table 2, Eq. (7), Fig. 1(a)). Based on the error analysis (Table 3), Eq. (7) is reasonably accurate and is applicable to concrete strengths ranging from 4 to 120 MPa, (580 to 17,400 psi) regardless of mixture proportions, the nature of the cementitious materials, curing time, and curing temperature;
2. The ratio of the tensile strength to compressive strength \( f_{spl/p} \) is influenced by the level of concrete strength. At low compressive strengths, the splitting tensile strengths are as high as 10% of the cylinder compressive strength but at extremely higher compressive strengths, the ratio reduces to approximately 5% (Fig. 1);

3. The commonly accepted 0.5 power relationship between the splitting tensile strength and cylinder compressive strength was not determined to be realistic; thus the ACI model should be re-evaluated for high-strength concrete; and

4. It is demonstrated that, knowing only the cylinder compressive strength of concrete, Johnston’s strength criterion (Eq. (8)) and the ratio of the splitting tensile to compressive strength (Eq. (7)) can be used to estimate the axial compressive strength of concrete under confining stress without performing triaxial tests (Table 5 and Fig. 3).

ACKNOWLEDGMENTS

The authors are greatly indebted to the members of the board, Yapı Merkezi Inc., Istanbul, Turkey, for their close attention and encouragement throughout the course of this study. E. Arçılıoğlu, Honorary Chair of Yapı Merkezi Inc., is gratefully acknowledged for his academic interest and support. The assistance of Ö. S. Köylüoğlu is also appreciated.

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DISCUSSION

Evaluation of Ratio between Splitting Tensile Strength and Compressive Strength for Concretes up to 120 MPa and its Application in Strength Criterion. Paper by Nihal Arıoğlu, Z. Canan Girgin, and Ergin Arıoğlu

Discussion by Nabi Yüzer
Associate Professor of Civil Engineering, Yıldız Technical University, Istanbul, Turkey.

The authors' evaluations on the ratio of splitting tensile strength to cylinder compressive strength of concrete as a function of compressive strength in a large scale are appreciated. In this study, it is noted that Eq. (7) is applicable to concrete with various mixture proportions, cementitious materials with and without silica fume, and 60 concrete samples with and without ground granulated blast-furnace slag under chloride effect.

In other words, the discussor put the compressive strength values of 132 concrete samples in total into Eq. (7) to predict splitting tensile strength values. It could be seen in Fig. A, however, that experimentally measured splitting tensile strength values (which were published in References 39 and 40) deviate dramatically from the predicted ones found by using Eq. (7) recommended by the authors. Consequently, in evaluating the quality of concrete and/or reinforced concrete exposed to chloride effect, the empirical equations for the tensile strength consisting only of the compressive strength as a material parameter would not be valid; the tensile strength should be tested separately.

NOTATION

\[ f_c = \text{cylinder compressive strength} \]
\[ f_{sc} = \text{splitting tensile strength} \]

REFERENCES
chloride ion, use of any of the relationships in which the splitting strength is estimated on the basis of the compressive strength will considerably overestimate the splitting strength of concrete. Recalling the study of Rezanoff and Corbett, a chloride-based setting and accelerating admixture reduces the splitting tensile strength as compared with normal—no accelerated—concrete. In this discussion, our equation, Eq. (7), introduced a modification factor \( \alpha \), to account for the effect of chloride on the tensile strength of concrete as displayed in Fig. B. The value of \( \alpha \) was obtained by means of least-squares regression analysis. For this analysis, the data were gathered from Rezanoff and Corbett's and Issa et al.'s studies. According to the analysis, the statistical values of \( \alpha \) were determined as the average \( \bar{\alpha} \) = 0.84, the standard deviation \( \sigma = 0.05 \), and the coefficient of variation \( V = 100 \times \bar{\alpha} \times 0.05 \). From the value of \( V \), it can be concluded that there is a very limited variation in the value of \( \bar{\alpha} \). In other words, the mean \( \bar{\alpha} \) of 0.84 can be taken as “constant of proportionality” for Eq. (7) correlating the splitting tensile strength with the compressive strength of concrete subjected to chloride effect.

In passing, it should be mentioned that the influence of strength range, curing condition, time, and concentration of chloride ion on the value of \( \bar{\alpha} \) are still unknown. In brief, for the assessment of the influences in question, the detailed experimental test program and statistical analysis are required.

To evaluate the accuracy of the modified Eq. (7) against experimental results, the data (Table A) reported by Yüzer and Aköz were used as control data. The splitting tensile strengths predicted from the equation under review are compared with the experimental data in Fig. C by making use of the 1:1 technique. The average derivations (\( \Delta = 10.1 \% ; \delta = 7.7 \% \)) obtained can be considered to be acceptable.

**Reply to Kenneth W. Day**

It is well known from the literature that the aggregate characteristics (type, particle shape, surface texture) have an important effect on the mechanical properties. In particular, the bond between the aggregate and the paste is a key factor affecting the tensile strength of concrete. This mechanical interlocking, or bond strength, can be enhanced by using clean aggregates with angular shape and rough texture. Hence, the tensile strength of concrete can be increased at a

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**Table A—Compressive and splitting tensile strengths of concrete subjected to chloride effect**

<table>
<thead>
<tr>
<th>Specimen code</th>
<th>Specimen code</th>
<th>Specimen code</th>
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<th>Specimen code</th>
<th>Specimen code</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(28 + 28 days)</td>
<td>(28 + 90 days)</td>
<td>(28 + 180 days)</td>
<td>(28 + 28 days)</td>
<td>(28 + 90 days)</td>
<td>(28 + 180 days)</td>
<td>(28 + 28 days)</td>
</tr>
<tr>
<td>SF0W02</td>
<td>28.4</td>
<td>28.2</td>
<td>28.2</td>
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</tr>
<tr>
<td>SF1W02</td>
<td>26.3</td>
<td>26.3</td>
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</tr>
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<tr>
<td>SF1W1</td>
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<tr>
<td>SF2W1</td>
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<td>SF0W3</td>
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<td>31.2</td>
<td>31.2</td>
<td>31.2</td>
<td>31.2</td>
</tr>
</tbody>
</table>

Notes: SF0 equals mixes without silica fume; SF1 equal mixes with 10% and 20% silica fume by weight of concrete; W0 equals specimens kept in only water (20 ± 3 °C); W1, W2 equal specimens kept in water containing various CI{sup}- chloride ion concentration (1500 mg/L, 3000 mg/L, and 4000 mg/L); and W3 equals solution prepared at rate of 40000 mg/L, chloride ion concentration. The size of specimens used in compressive and splitting tension tests were 100 x 200 mm x 4 in cylinder. (The value of 0.93/0.85 as conversion factors of splitting tensile strength determined on 4 in x 8 in cylinder specimen to strength of 150 x 300 cylinder specimen was applied.)
aggregate of rounded/irregular shape and smooth texture—uncrushed gravel—maximum aggregate size $D_{max}$ = 19 mm, various curing ages. The compressive strengths of 150 mm cubes were converted to the strength of $Φ 150 \times 300$ mm cylinder via the conversion factor of $0.8^{24}$. From Fig. D it is evident that, for a given compressive strength (10 MPa $\leq f_c \leq 50$ MPa), the ratio of $f_{spf/c}$ for concretes with uncrushed gravels results in a reduction of almost 16% compared to the concretes containing crushed rock aggregates. Briefly, in the concretes with uncrushed gravel for a given compressive strength, there is a decrease in the splitting tensile strength. Its main reason is the weaker bond developed between the aggregate and the surrounding hydrated cement paste.

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Disc. 103-M96/From the Jan.-Feb. 2006 ACI Materials Journal, p. 60

Influence of Concrete Material Ductility on Shear Response of Stud Connections. Paper by Shunzhi Qian and Victor C. Li

Discussion by Shiming Chen

Professor, School of Civil Engineering, Tongji University, Shanghai, China.

The discusser appreciates the authors' comprehensive work to investigate the potential application of engineered cementitious composites (ECC) in shear stud connections for steel-concrete composite beams with a desired ductile slip capacity. Some test findings that are interesting to the discusser, however, were not well clarified when the test results were compared with the predictions based on the design method (AASHTO LRFD). Discussions are drawn as follows:

**Compressive strength of concrete**

Accordingly, it is understood that $f'_{c}$, the compressive strength of concrete in Table 2, should be the cylinder compressive strength. For a comparison, the measured and predicted strength per stud based on AASHTO equations were drawn in Fig. A.

It is found that the predicted strengths based on AASHTO LRFD method were all greater than the measured strengths for concrete and RC stud connections so that the method would be unsafe, which is argued by the discusser. As being noted that a steel reinforcement ratio of 0.86% had been used for transverse reinforcement in RC specimens that could prevent earlier longitudinal splitting shear failure in the connections, is $f'_{c}$ adopted in the paper a cube compressive strength rather than the cylinder compressive strength?

**Strength and failure modes**

In design practice, the shear stud connections are normally classified as ductile as far as ratio $h/d > 4$, where $h$ and $d$ are the overall height and diameter of a stud, respectively. Ductile behaviors were observed in RC, SFRC, and ECC specimens, except for concrete specimens.